1. [3] Compute:
\[ \lim_{x \to 0} \frac{x^2}{1 - \cos(x)} \]

2. [3] Determine the real number \( a \) having the property that \( f(a) = a \) is a relative minimum of \( f(x) = x^4 - x^3 - x^2 + ax + 1 \).

3. [4] Let \( a \) be a positive real number. Find the value of \( a \) such that the definite integral
\[ \int_a^2 \frac{dx}{x + \sqrt{x}} \]
achieves its smallest possible value.

4. [4] Find the real number \( \alpha \) such that the curve \( f(x) = e^x \) is tangent to the curve \( g(x) = \alpha x^2 \).

5. [5] The function \( f : \mathbb{R} \to \mathbb{R} \) satisfies \( f(x^2)f''(x) = f'(x)f(x^2) \) for all real \( x \). Given that \( f(1) = 1 \) and \( f''(1) = 8 \), determine \( f'(1) + f''(1) \).

6. [5] The elliptic curve \( y^2 = x^3 + 1 \) is tangent to a circle centered at \((4, 0)\) at the point \((x_0, y_0)\). Determine the sum of all possible values of \( x_0 \).

7. [5] Compute
\[ \sum_{n=1}^{\infty} \frac{1}{n \cdot (n+1) \cdot (n+1)!} \]

8. [6] Suppose that \( \omega \) is a primitive 2007\(^{th} \) root of unity. Find \( (2^{2007} - 1) \sum_{j=1}^{2006} \frac{1}{2 - \omega^j} \).

For this problem only, you may express your answer in the form \( m \cdot n^k + p \), where \( m, n, k, \) and \( p \) are positive integers. Note that a number \( z \) is a primitive \( n \)\(^{th} \) root of unity if \( z^n = 1 \) and \( n \) is the smallest number amongst \( k \) = 1, 2, \ldots, \( n \) such that \( z^k = 1 \).

9. [7] \( g \) is a twice differentiable function over the positive reals such that
\[ g(x) + 2x^3g'(x) + x^4g''(x) = 0 \quad \text{for all positive reals } x. \tag{1} \]
\[ \lim_{x \to \infty} xg(x) = 1 \tag{2} \]

Find the real number \( \alpha > 1 \) such that \( g(\alpha) = 1/2 \).

10. [8] Compute
\[ \int_0^\infty \frac{e^{-x} \sin(x)}{x} \, dx \]