1. [20] Consider a regular $n$-gon with $n > 3$, and call a line acceptable if it passes through the interior of this $n$-gon. Draw $m$ different acceptable lines, so that the $n$-gon is divided into several smaller polygons.

(a) Prove that there exists an $m$, depending only on $n$, such that any collection of $m$ acceptable lines results in one of the smaller polygons having 3 or 4 sides.

(b) Find the smallest possible $m$ which guarantees that at least one of the smaller polygons will have 3 or 4 sides.

2. [25] 2014 triangles have non-overlapping interiors contained in a circle of radius 1. What is the largest possible value of the sum of their areas?

3. [30] Fix positive integers $m$ and $n$. Suppose that $a_1, a_2, \ldots, a_m$ are reals, and that pairwise distinct vectors $v_1, \ldots, v_m \in \mathbb{R}^n$ satisfy

$$\sum_{j \neq i} a_j \frac{v_j - v_i}{\|v_j - v_i\|^3} = 0$$

for $i = 1, 2, \ldots, m$.

Prove that

$$\sum_{1 \leq i < j \leq m} \frac{a_i a_j}{\|v_j - v_i\|^2} = 0.$$

4. [35] Let $\omega$ be a root of unity and $f$ be a polynomial with integer coefficients. Show that if $|f(\omega)| = 1$, then $f(\omega)$ is also a root of unity.

5. [40] Let $n$ be a positive integer, and let $A$ and $B$ be $n \times n$ matrices with complex entries such that $A^2 = B^2$. Show that there exists an $n \times n$ invertible matrix $S$ with complex entries that satisfies $S(AB - BA) = (BA - AB)S$. 