1. Assume we have 4 colors - 1, 2, 3, and 4. Fix the bottom as color 1. On the remaining
sides you can have colors 2, 3, 4 (in that order), or 2, 4, 3, which are not rotationally
identical. So, there are 2 ways to color it.

2. Since \( \cos \frac{2\pi}{3} = -\frac{1}{2} \) and \( \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2} \), we can write the first term as \( (\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})^6 \).
   Since \( \cos \frac{4\pi}{3} = -\frac{1}{2} \) and \( \sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2} \), we can write the second term as \( (\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})^6 \).
   Now, we apply DeMoivre’s Theorem to simplify the first expression to \( (\cos 6 \cdot \frac{2\pi}{3} + \sin 6 \cdot \frac{2\pi}{3}) = (\cos 4\pi + \sin 4\pi) = 1 \). Similarly, we simplify the second expression to \( (\cos 6 \cdot \frac{4\pi}{3} + \sin 6 \cdot \frac{4\pi}{3}) = (\cos 8\pi + \sin 8\pi) = 1 \). Thus, the total sum is \( 1 + 1 = 2 \).

3. We know that \( \frac{1}{n^2+2n} = \frac{1}{n(n+2)} = \frac{\frac{1}{n} - \frac{1}{n+2}}{2} \). So, if we sum this from 1 to \( \infty \), all terms except for \( \frac{1}{2} + \frac{1}{2} \) will cancel out (a "telescoping" series). Therefore, the sum will be \( \frac{3}{4} \).

4. The possibilities for the numbers are:
   - all five are divisible by 3
   - three are divisible by 3, one is \( \equiv 1 \pmod{3} \) and one is \( \equiv 2 \pmod{3} \)
   - two are divisible by 3, and the other three are either \( \equiv 1 \pmod{3} \) or \( \equiv 2 \pmod{3} \)
   - one is divisible by 3, two are \( \equiv 1 \pmod{3} \) and two are \( \equiv 2 \pmod{3} \)
   - four are \( \equiv 1 \pmod{3} \) and one is \( \equiv 2 \pmod{3} \)
   - four are \( \equiv 2 \pmod{3} \) and one is \( \equiv 1 \pmod{3} \)

   This gives us 1001 possible combinations out of \( \binom{15}{5} \) or 3003. So, the probability is \( \frac{1001}{3003} = \frac{1}{3} \).

5. **153,370,371,407**

6. There are 6 people that could get their hat back, so we must multiply 6 by the number of ways that the other 5 people can arrange their hats such that no one gets his/her hat back. So, the number of ways this will happen is \( (6 \cdot \text{derangement of 5}) \), or \( 6 \cdot 44 = 264 \). Since there are \( 6! = 720 \) possible arrangements of hats, the probability of exactly one person getting their hat back is \( \frac{264}{720} = \frac{11}{30} \).

7. We can view these conditions as a geometry diagram as seen below. So, we know that
   \( \frac{e}{f} = \frac{3}{7} \) (since \( e = a - b = \frac{3}{7}c - \frac{3}{7}d = \frac{3}{7}f \) and we know that \( \sqrt{e^2 + f^2} = 15 \) (since this is
   \( \sqrt{a^2 + c^2 - \sqrt{b^2 + d^2}} \)). Also, note that \( ac + bd - ad - bc = (a - b)(c - d) = ef \). So, solving
   for \( e \) and \( f \), we find that \( e^2 + f^2 = 225 \), so \( 16e^2 + 16f^2 = 3600 \), so \( (4e)^2 + (4f)^2 = 3600 \),
   so \( (3f)^2 + (4f)^2 = 3600 \), so \( f^2(3^2 + 4^2) = 3600 \), so \( 25f^2 = 3600 \), so \( f^2 = 144 \) and \( f = 12 \).
   Thus, \( e = \frac{3}{7} \cdot 12 = 9 \). Therefore, \( ef = 9 \cdot 12 = 108 \).
8. It suffices to consider the complements of the graphs, so we are looking for graphs with 9 vertices, where each vertex is connected to 2 others. There are 4 different graphs - see below.

9. The probability of the Reals hitting 0 singles is \( \left( \frac{4}{9} \right)^3 \). The probability of the Reals hitting exactly 1 single is \( \binom{3}{2} \cdot \left( \frac{2}{9} \right)^3 \cdot \frac{1}{3} \), since there are 3 spots to put the two outs (the last spot must be an out, since the inning has to end on an out). The probability of the Reals hitting exactly 2 singles is \( \binom{2}{2} \cdot \left( \frac{2}{3} \right)^3 \cdot \left( \frac{1}{3} \right)^3 \). If any of these happen, the Alphas win right away. Adding these gives us a \( \frac{666}{729} \) chance of this happening. If exactly 4 singles occur (with probability \( \binom{6}{2} \cdot \left( \frac{2}{9} \right)^3 \cdot \left( \frac{1}{3} \right)^4 \)), then there is a \( \frac{2}{9} \) chance that the Alphas win. The probability of this happening is \( \frac{2}{9} \cdot \frac{10}{729} \). Thus, the total probability of the Alphas winning is the sum of these two probabilities, or \( \frac{666}{729} + \frac{10}{729} = \frac{234}{243} \).

10. A will say yes when B says no to \( n - 1 \) or \( n \), as A will then know B’s number is one greater than A’s number. Thus, A responds first, after \( \frac{n-1}{2} \) ”no” responses if \( n \) is odd, after \( \frac{n}{2} \) ”no” responses if \( n \) is even.