1st Annual Harvard-MIT November Tournament
Saturday 8 November 2008

Individual Round

1. [2] Find the minimum of $x^2 - 2x$ over all real numbers $x$.
2. [3] What is the units digit of $7^{2009}$?
3. [3] How many diagonals does a regular undecagon (11-sided polygon) have?
4. [4] How many numbers between 1 and 1,000,000 are perfect squares but not perfect cubes?
5. [5] Joe has a triangle with area $\sqrt{3}$. What’s the smallest perimeter it could have?
6. [5] We say “$s$ grows to $r$” if there exists some integer $n > 0$ such that $s^n = r$. Call a real number $r$ “sparse” if there are only finitely many real numbers $s$ that grow to $r$. Find all real numbers that are sparse.
7. [6] Find all ordered pairs $(x, y)$ such that
   \[(x - 2y)^2 + (y - 1)^2 = 0.\]
8. [7] How many integers between 2 and 100 inclusive cannot be written as $m \cdot n$, where $m$ and $n$ have no common factors and neither $m$ nor $n$ is equal to 1? Note that there are 25 primes less than 100.
9. [7] Find the product of all real $x$ for which
   \[2^{3x+1} - 17 \cdot 2^{2x} + 2^{x+3} = 0.\]
10. [8] Find the largest positive integer $n$ such that $n^3 + 4n^2 - 15n - 18$ is the cube of an integer.