# HMMT November 2016

**November 12, 2016**

## Theme Round

1. DeAndre Jordan shoots free throws that are worth 1 point each. He makes 40% of his shots. If he takes two shots find the probability that he scores at least 1 point.

2. Point $P_1$ is located 600 miles West of point $P_2$. At 7:00 AM a car departs from $P_1$ and drives East at a speed of 50 miles per hour. At 8:00 AM another car departs from $P_2$ and drives West at a constant speed of $x$ miles per hour. If the cars meet each other exactly halfway between $P_1$ and $P_2$, what is the value of $x$?

3. The three points $A$, $B$, $C$ form a triangle. $AB = 4$, $BC = 5$ $AC = 6$. Let the angle bisector of $\angle A$ intersect side $BC$ at $D$. Let the foot of the perpendicular from $B$ to the angle bisector of $\angle A$ be $E$. Let the line through $E$ parallel to $AC$ meet $BC$ at $F$. Compute $DF$.

4. A positive integer is written on each corner of a square such that numbers on opposite vertices are relatively prime while numbers on adjacent vertices are not relatively prime. What is the smallest possible value of the sum of these 4 numbers?

5. Steph Curry is playing the following game and he wins if he has exactly 5 points at some time. Flip a fair coin. If heads, shoot a 3-point shot which is worth 3 points. If tails, shoot a free throw which is worth 1 point. He makes $\frac{1}{2}$ of his 3-point shots and all of his free throws. Find the probability he will win the game. (Note he keeps flipping the coin until he has exactly 5 or goes over 5 points)

6. Let $P_1, P_2, \ldots, P_6$ be points in the complex plane, which are also roots of the equation $x^6+6x^3-216 = 0$. Given that $P_1P_2P_3P_4P_5P_6$ is a convex hexagon, determine the area of this hexagon.

7. Seven lattice points form a convex heptagon with all sides having distinct lengths. Find the minimum possible value of the sum of the squares of the sides of the heptagon.

8. Let $P_1P_2\ldots P_8$ be a convex octagon. An integer $i$ is chosen uniformly at random from 1 to 7, inclusive. For each vertex of the octagon, the line between that vertex and the vertex $i$ vertices to the right is painted red. What is the expected number times two red lines intersect at a point that is not one of the vertices, given that no three diagonals are concurrent?

9. The vertices of a regular nonagon are colored such that 1) adjacent vertices are different colors and 2) if 3 vertices form an equilateral triangle, they are all different colors.

   Let $m$ be the minimum number of colors needed for a valid coloring, and $n$ be the total number of colorings using $m$ colors. Determine $mn$. (Assume each vertex is distinguishable.)

10. We have 10 points on a line $A_1, A_2 \cdots A_{10}$ in that order. Initially there are $n$ chips on point $A_1$. Now we are allowed to perform two types of moves. Take two chips on $A_i$, remove them and place one chip on $A_{i+1}$, or take two chips on $A_{i+1}$, remove them, and place a chip on $A_{i+2}$ and $A_i$ . Find the minimum possible value of $n$ such that it is possible to get a chip on $A_{10}$ through a sequence of moves.